The Spin Structure of the Nucleon

– part 1 –

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Spin and Symmetries – Praha 2004

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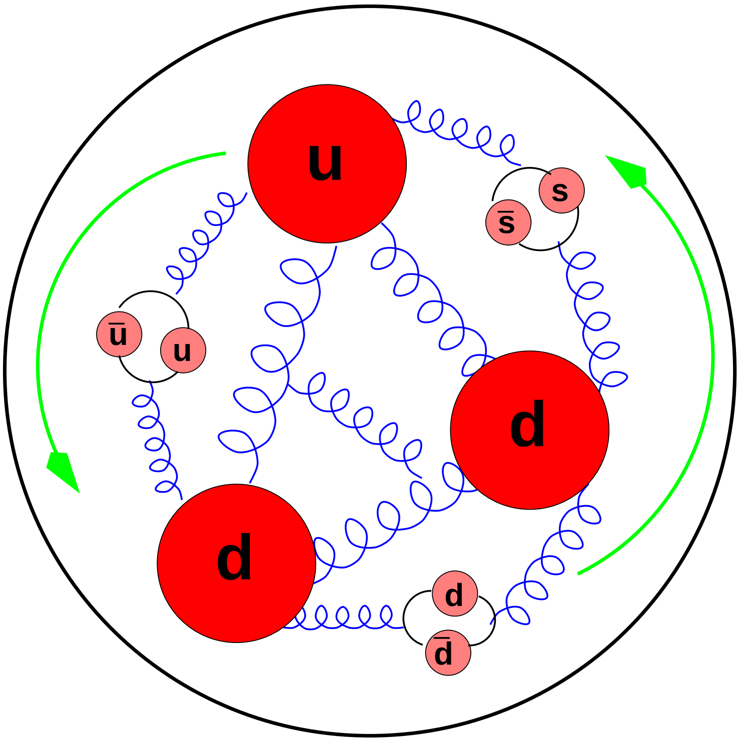
PROTON SPIN PUZZLE



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

quarks antiquarks — gluons — angular-momentum

- $\Delta\Sigma = \Delta u + \Delta d + \Delta s$
- **naive quark model** $\Delta\Sigma = 1$
- **EMC (1985)** $\Delta\Sigma << 1$
  $\Delta s \approx -0.1$
- **SMC, SLAC, HERMES** (1988-2000) $\Delta\Sigma \approx 0.2$

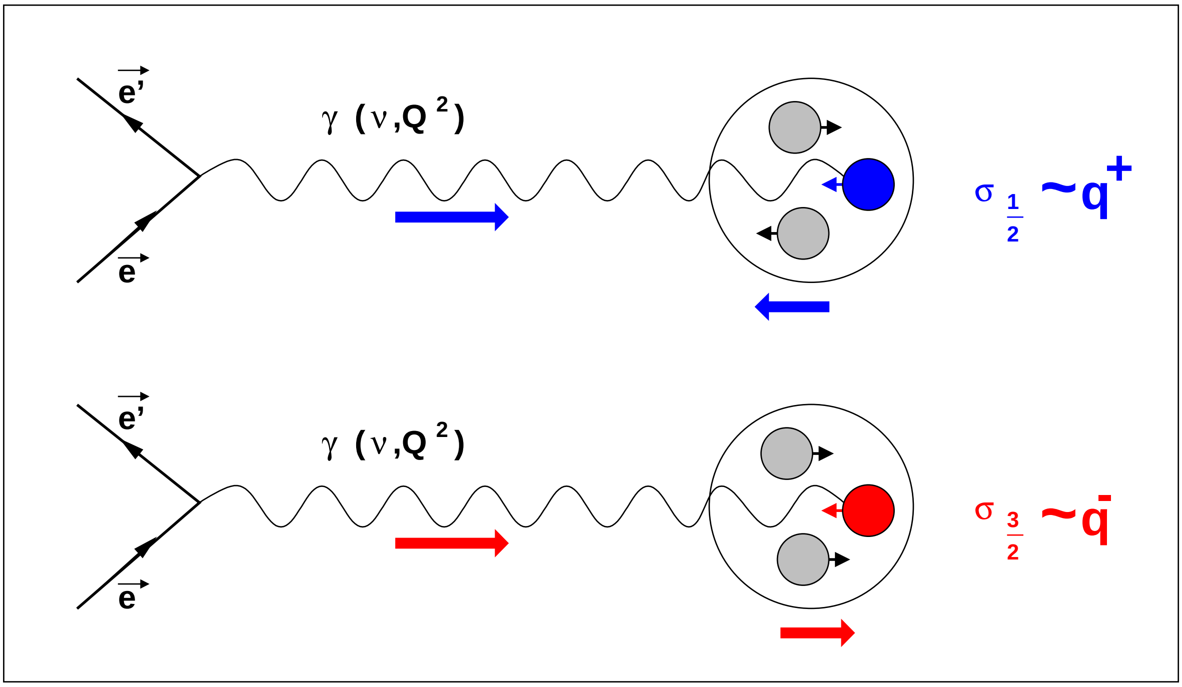
CONTENTS

- Polarized DIS
- Experimental method
- Asymmetries A_1
- Structure functions
- QCD-Analysis
- Sum rules
- Duality

METHOD

$$\vec{\gamma}\ \vec{\mathrm{N}} \longrightarrow \mathrm{X}$$

- **Absorption of polarised photons**



- **Photon nucleon asymmetry**

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \approx \frac{\Sigma_q\ e_q^2\ (q(x)^+ - q(x)^-)}{\Sigma_q\ e_q^2\ (q(x)^+ + q(x)^-)} = \frac{g_1(x)}{F_1(x)}$$

q^+, q^-, $\Delta q = q^+ - q^-$ quark distributions

$g_1(x)$ spin structure function

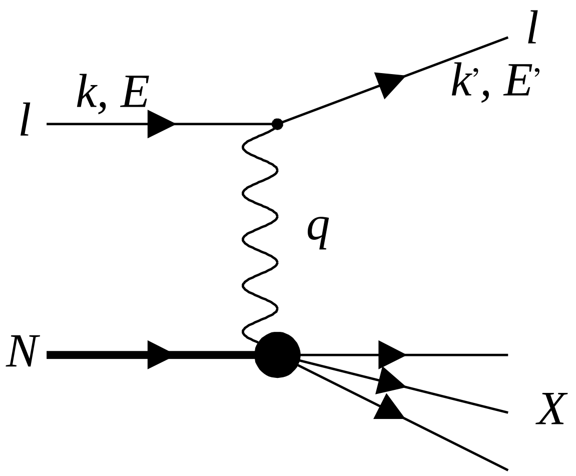
- **Lepton nucleon asymmetry**

$$A_{\parallel}(x, Q^2) = \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow} + d\sigma^{\uparrow\uparrow}} \approx D\ A_1$$

$\uparrow$ lepton spin, $\uparrow$ nucleon spin

D depolarisation factor

POLARISED DIS



$$Q^2 = -q^2$$
$$\nu = E - E'$$
$$x = \frac{Q^2}{2M\nu}$$
$$y = \frac{\nu}{E}$$

- **Cross section**

$$\mathrm{d}\sigma = \mathrm{d}\bar{\sigma} \pm \frac{1}{2}\mathrm{d}\Delta\sigma$$

with

$$\mathrm{d}\bar{\sigma} = aF_1(x,Q^2) + bF_2(x,Q^2)$$
$$\mathrm{d}\Delta\sigma = \alpha g_1(x,Q^2) + \beta g_2(x,Q^2)$$

- ***F_1, F_2*** unpolarised structure functions
  g_1, g_2 spin structure functions

- **longitudinal pol.**: $\mathrm{d}\Delta\sigma_{\parallel} = \mathrm{d}\sigma^{\uparrow\downarrow} + \mathrm{d}\sigma^{\uparrow\uparrow}$
  transverse pol.: $\mathrm{d}\Delta\sigma_{\perp} = \mathrm{d}\sigma^{\uparrow\rightarrow} - \mathrm{d}\sigma^{\uparrow\leftarrow}$

- **Double spin symmetry**

$$A = \frac{\Delta\sigma}{2\bar{\sigma}}$$

EXPERIMENTAL METHOD

- **Longitudinal / transverse target polarisation**

$$A_{\parallel}(x,Q^2) = \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow} + d\sigma^{\uparrow\uparrow}} \qquad A_{\perp}(x,Q^2) = \frac{d\sigma^{\uparrow\rightarrow} - d\sigma^{\uparrow\leftarrow}}{d\sigma^{\uparrow\rightarrow} + d\sigma^{\uparrow\leftarrow}}$$

- Virtual photons are only partially polarized

$$= D(A_1 + \eta A_2) \qquad = d(A_2 - \eta' A_1)$$

D, d depolarisation factor: η, η' kinematic factors

- **Photon nucleon asymmetries A_1 and A_2**

$$A_1(x,Q^2) = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \qquad A_2(x,Q^2) = \frac{2\sigma_{LT}}{\sigma_{1/2} + \sigma_{3/2}}$$

- **Structure functions g_1 and g_2**

$$= \frac{1}{F_1}(g_1 - \gamma^2 g_2) \qquad = \frac{\gamma}{F_1}(g_1 + g_2)$$

- Limits:
  $|A_1| \leq 1$ $\qquad$ $|A_2| \leq \sqrt{R}$
- $\gamma = \frac{2Mx}{Q^2}$ small for SMC kinematics:

$$A_1 \approx g_1/F_1 \approx A_{\parallel}/D$$

EXPERIMENTS

SLAC	**E80, E130**	$\vec{e}\ \vec{p}$	≤ 20 GeV
CERN	**EMC**	$\vec{\mu}\ \vec{p}$	100–200 GeV
SLAC	**E142, 143**	$\vec{e}\ \vec{p}, \vec{n}, \vec{d}$	≤ 28 GeV
CERN	**SMC**	$\vec{\mu}\ \vec{p}, \vec{d}$	100, 190 GeV
SLAC	**E154, 155**	$\vec{e}\ \vec{p}, \vec{n}, \vec{d}$	≤ 50 GeV
DESY	**HERMES**	$\vec{e}\ \vec{p}, \vec{n}, \vec{d}$	27.5 GeV
CERN	**COMPASS**	$\vec{\mu}\ \vec{d}$	160 GeV
JLAB	**HALL A**	$\vec{e}\ \vec{n}$	6 GeV
JLAB	**CLAS**	$\vec{e}\ \vec{p}, \vec{d}$	6 GeV
JLAB	**RSS**	$\vec{e}\ \vec{p}, \vec{d}$	6 GeV

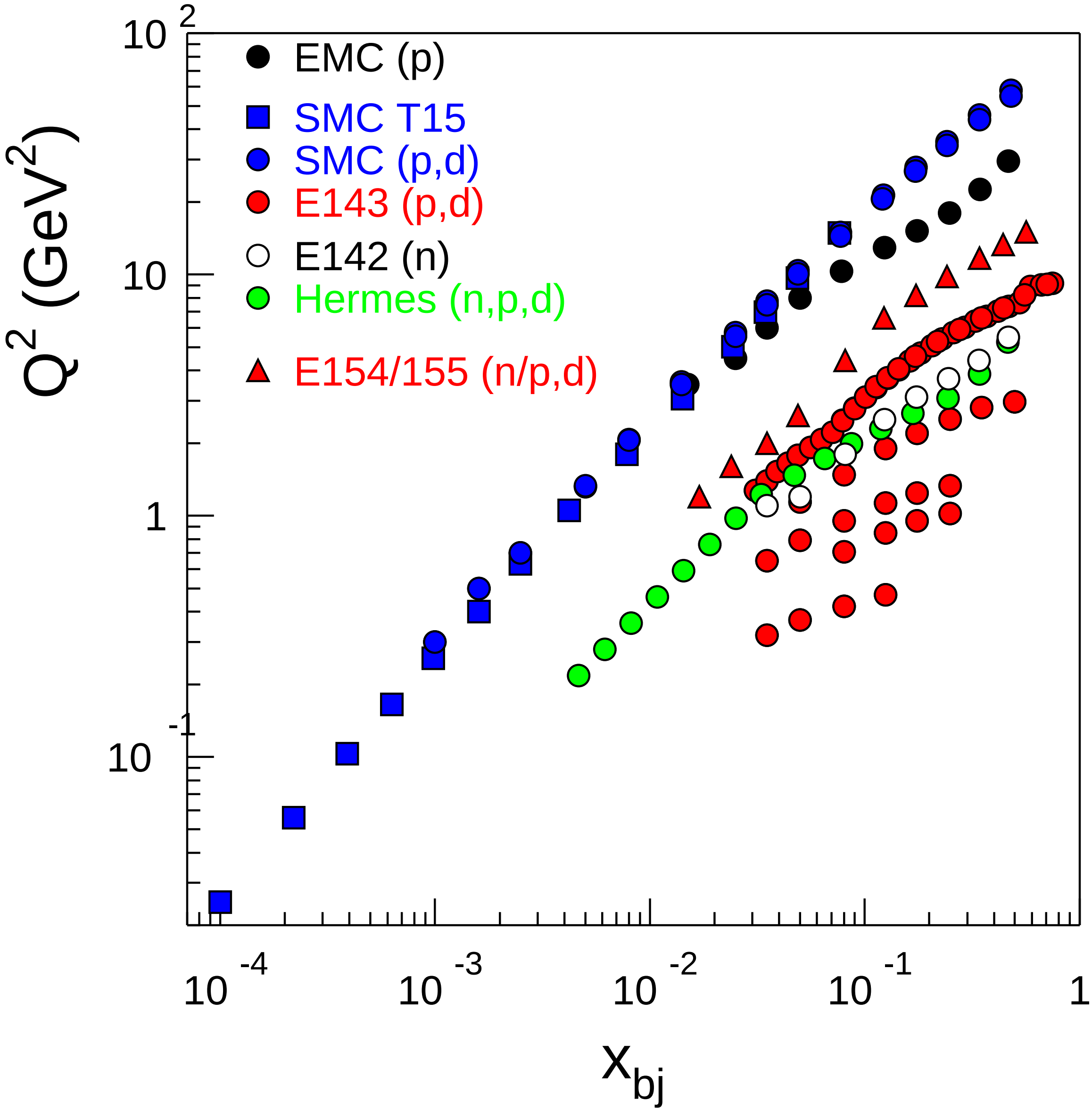
- **Experimental asymmetry**

$$A^{exp} = p_\mu\ p_T\ f\ D\ A_1 \sim 10^{-2}$$

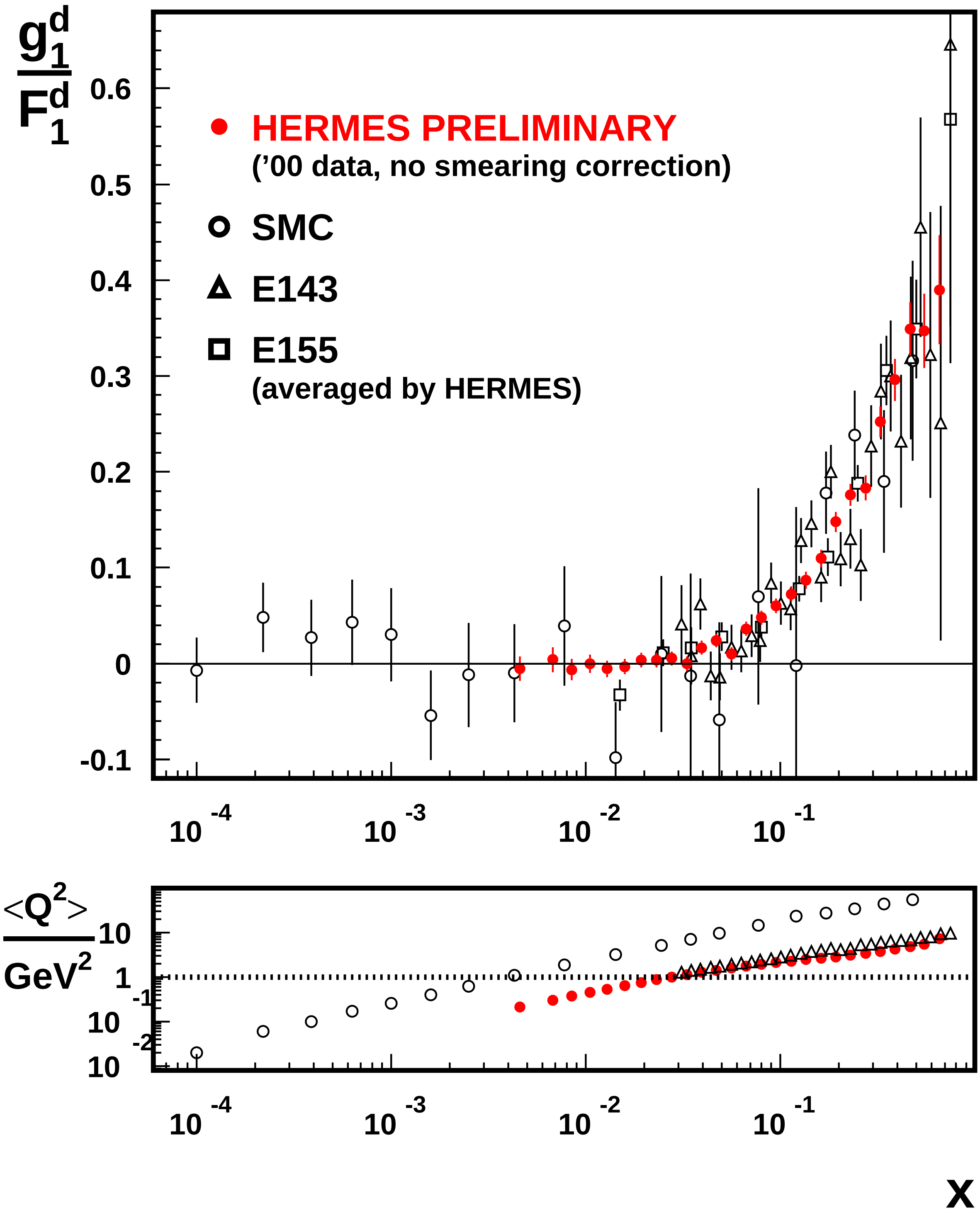
p_μ, p_T beam, target polarisation
f dilution factor

- Most experiments use solid state targets: $f \sim 0.1 - 0.4$
- HERMES uses storge cell: $f \approx 1$

KINEMATIC RANGE

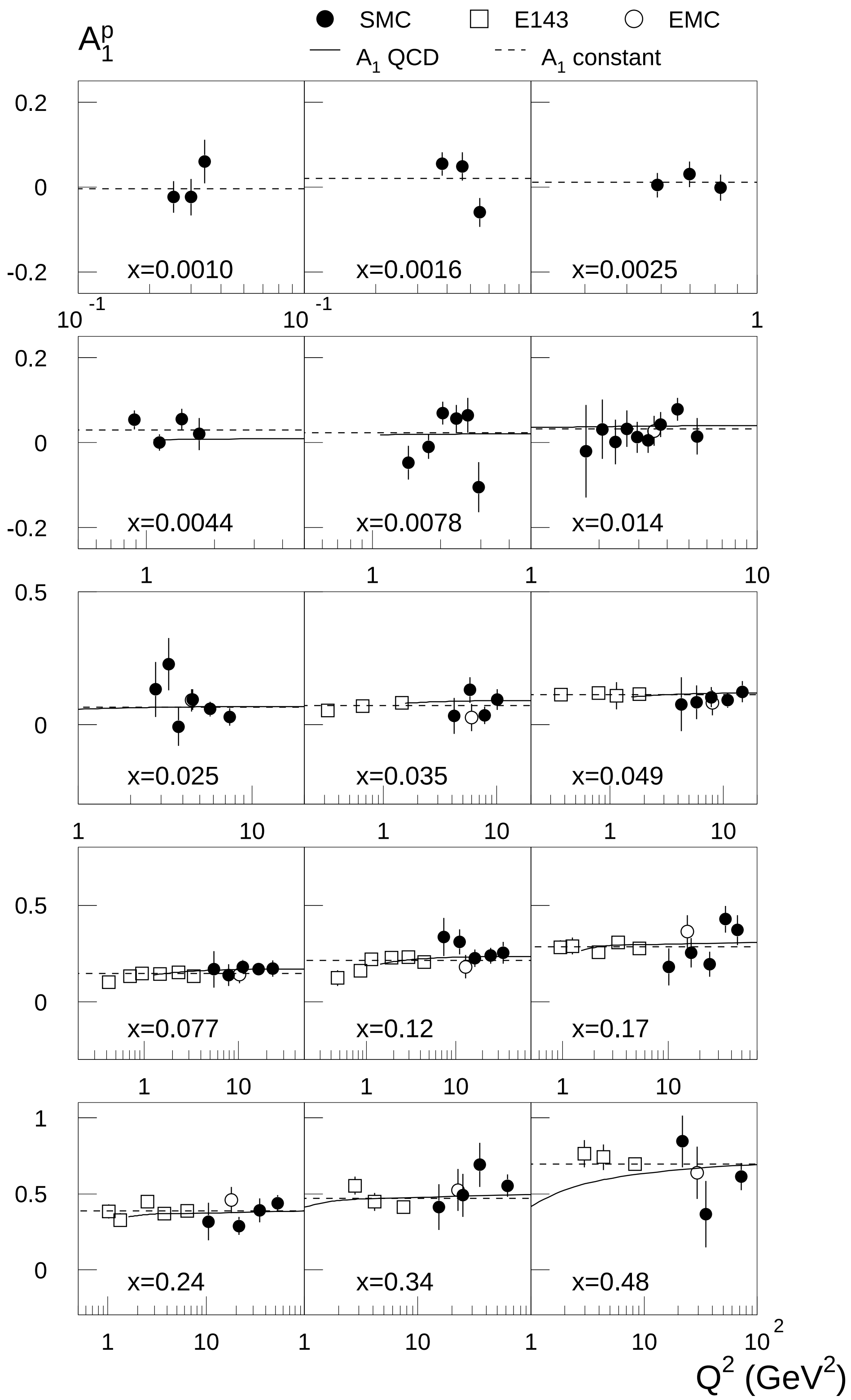


WORLD DATA FOR $A_1^d(x)$



- good statistical precision
- no Q^2 dependence visible

Q^2 DEPENDENCE OF A_1^p



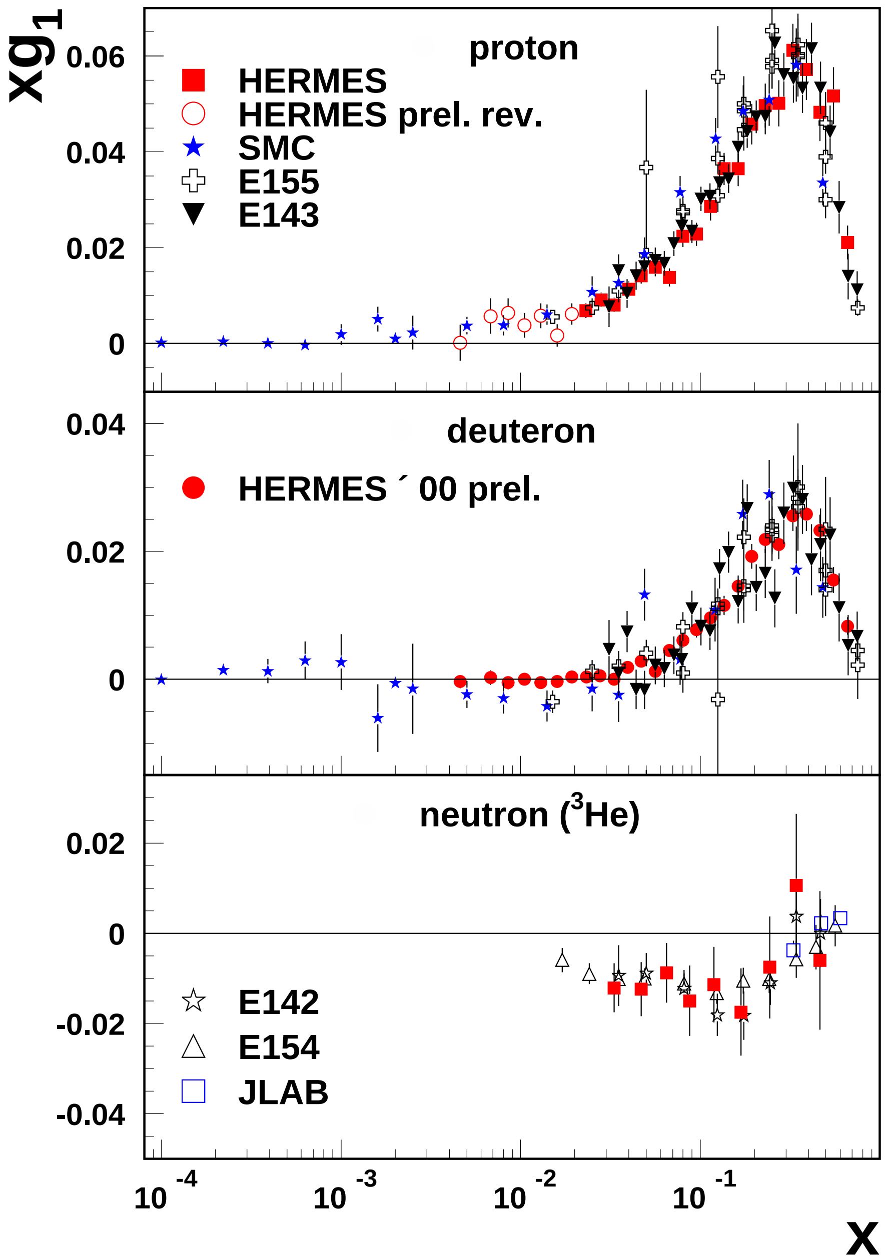
- No Q^2 dependence for $Q^2 > 1$ GeV^2 observed

xg_1 WORLD DATA

- **Spin structure function**

$$g_1 = \frac{1}{2}\Sigma_q\; e_q^2\; \Delta q(x) = A_1 \cdot \frac{F_2}{2x(1+R)}$$

- **Data:** SLAC, EMC, SMC, HERMES, JLAB



QCD CORRECTIONS

- **Polarized parton densities**

$$\Delta f = \begin{cases} \Delta q_{NS} \equiv \sum_{i=1}^{n_f} \left(e_i^2/\langle e^2\rangle - 1\right) \Delta q_i & \text{non singlet} \\ \Delta\Sigma \equiv \sum_{i=1}^{n_f} \Delta q_i & \text{singlet} \\ \Delta g & \text{gluon} \end{cases}$$

- **DGLAP evolution equations for Δf**

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} \Delta q_{NS} = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_{qq}^{NS} \otimes \Delta q_{NS}$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{pmatrix} \Delta P_{qq}^S & 2n_f \Delta P_{qg} \\ \Delta P_{gq} & \Delta P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}$$

- **Polarized splitting functions**

$\Delta P_{\mathrm{qq}} = P_{\mathrm{qq}}$ same Q^2 dep. for g_1 and F_1 at large x

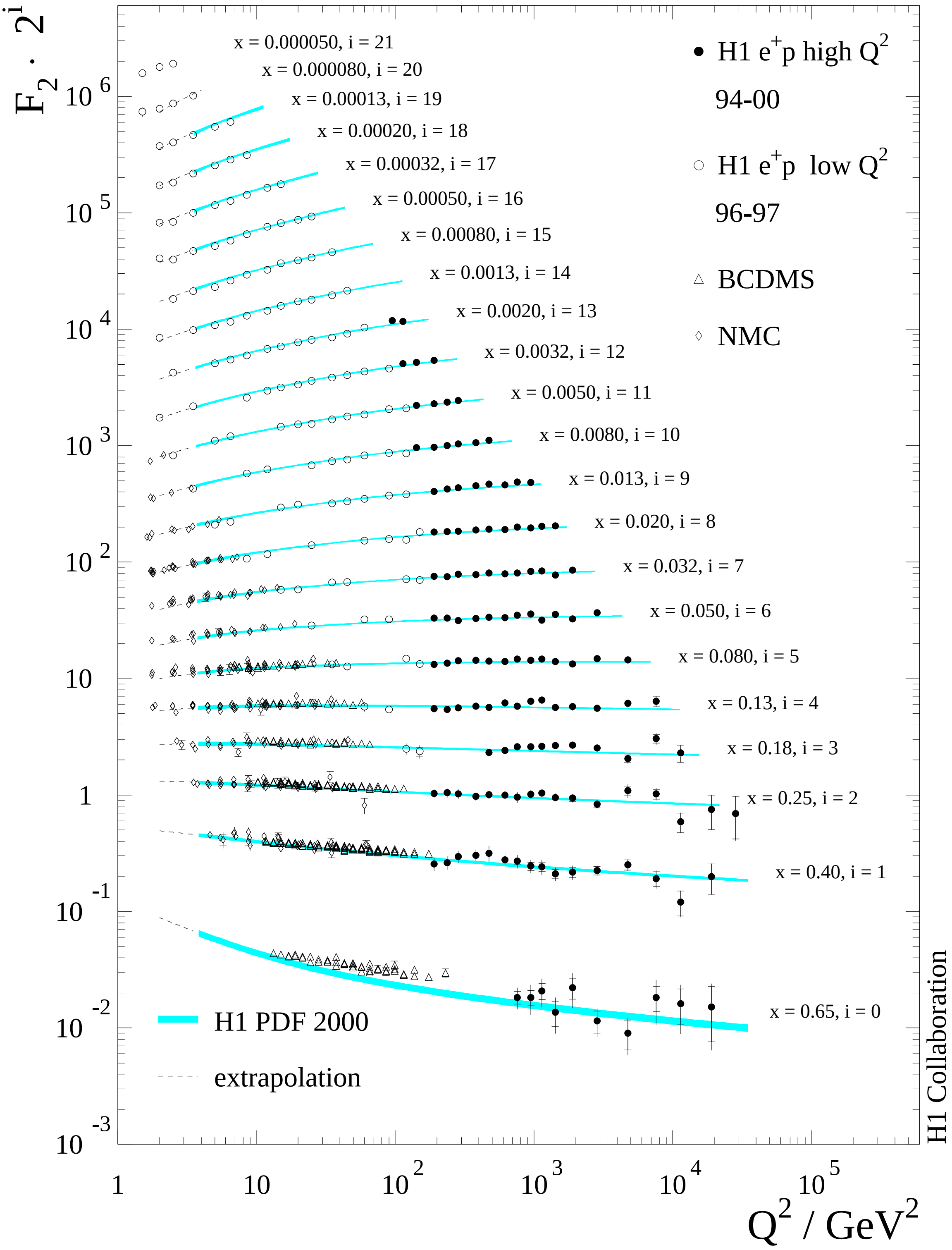
$\Delta P_{\mathrm{qg}} \neq P_{\mathrm{qg}}$ Q^2 dep. different

- **Spin structure function g_1**

$$g_1(x, Q^2) = \frac{1}{2}\langle e^2\rangle \left[C_{NS} \otimes \Delta q_{NS} + C_S \otimes \Delta\Sigma + 2n_f C_g \otimes \Delta g\right]$$

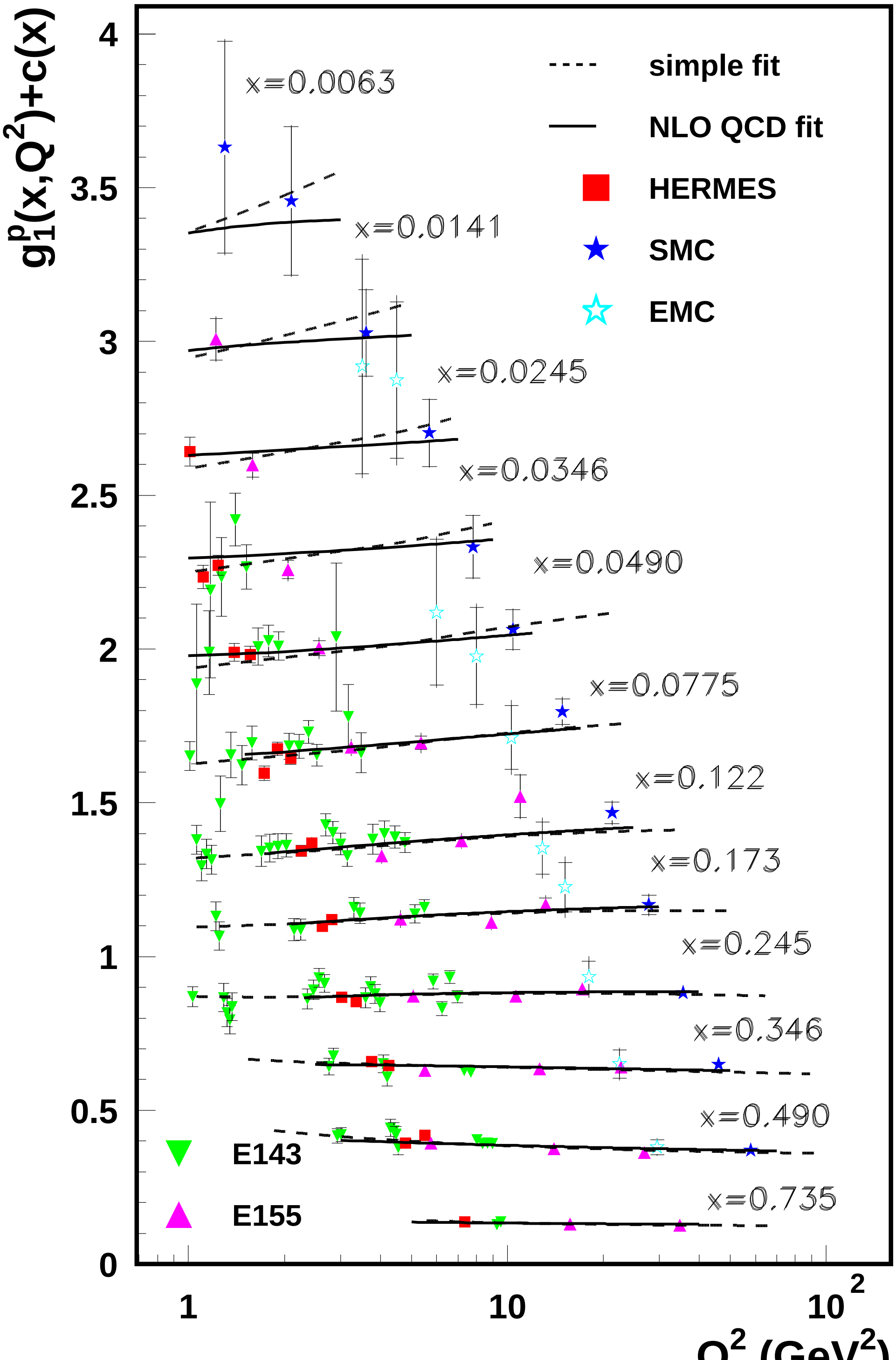
C coefficient functions

$F_2(x)$ VS. Q^2



F2 · 2^i
Q^2 / GeV^2
x = 0.000050, i = 21
x = 0.000080, i = 20
x = 0.00013, i = 19
x = 0.00020, i = 18
x = 0.00032, i = 17
x = 0.00050, i = 16
x = 0.00080, i = 15
x = 0.0013, i = 14
x = 0.0020, i = 13
x = 0.0032, i = 12
x = 0.0050, i = 11
x = 0.0080, i = 10
x = 0.013, i = 9
x = 0.020, i = 8
x = 0.032, i = 7
x = 0.050, i = 6
x = 0.080, i = 5
x = 0.13, i = 4
x = 0.18, i = 3
x = 0.25, i = 2
x = 0.40, i = 1
x = 0.65, i = 0
H1 e+p high Q2 94-00
H1 e+p low Q2 96-97
BCDMS
NMC
H1 PDF 2000
extrapolation
H1 Collaboration

$g_1(x)$ VS. Q^2



- much more limited kinematical range than for F_2
- QCD analysis needs some assumptions

QCD ANALYSIS

- **AAC** (asymmetry analysis collaboration) NLO analysis
- **Initial parametrisations** at $Q_0^2 = 1\ \text{GeV}^2$

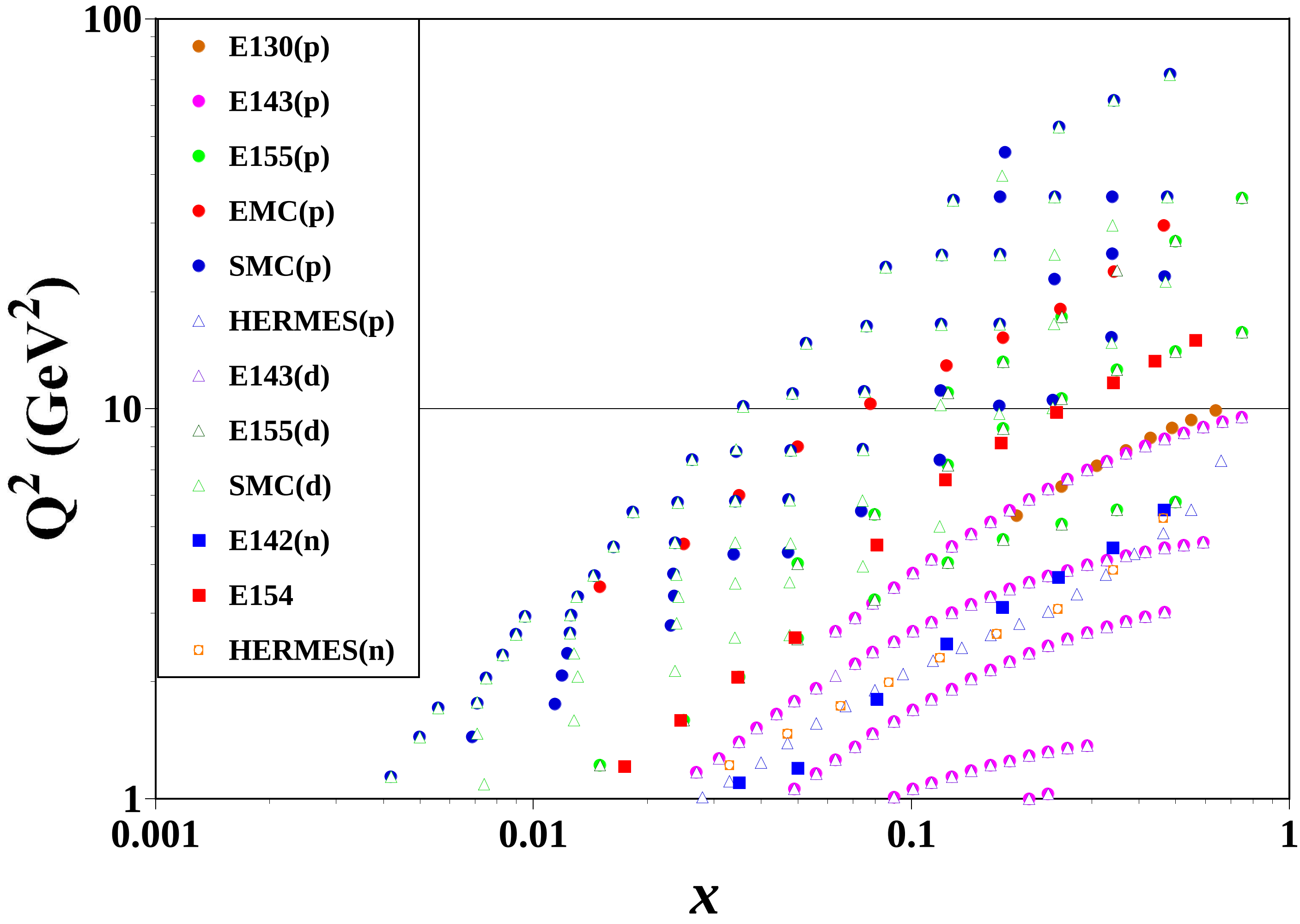
$$\Delta f_i(x, Q^2) = A_i\, x^{\alpha_i}\, (1 + \gamma_i x^{\lambda_i})\, f_i(x, Q_0^2)$$

$i = u_\text{V}, d_\text{V}, \bar{q}, g$
$A_i,\ \alpha_i,\ \gamma_i,\ \lambda_i$ parameters

- f_i unpolarised PDF (GRV98)
- $Q_0^2 = 1\ \text{GeV}^2$
- flavour symmetric sea: $\Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s}$
- correlation between $\Delta\bar{q}$ and Δg investigated in detail

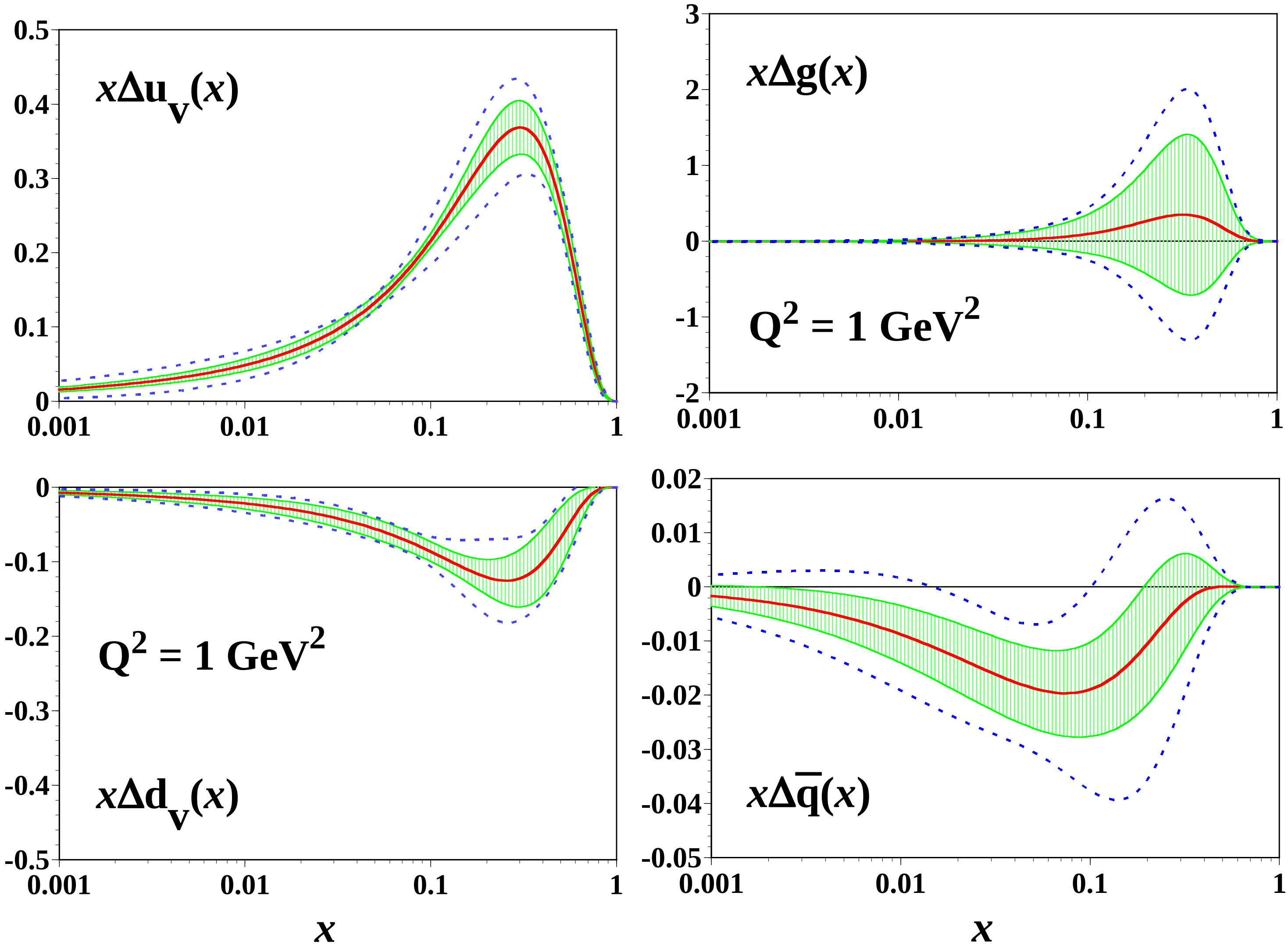
RECENT AAC ANALYSIS

- Data points used:



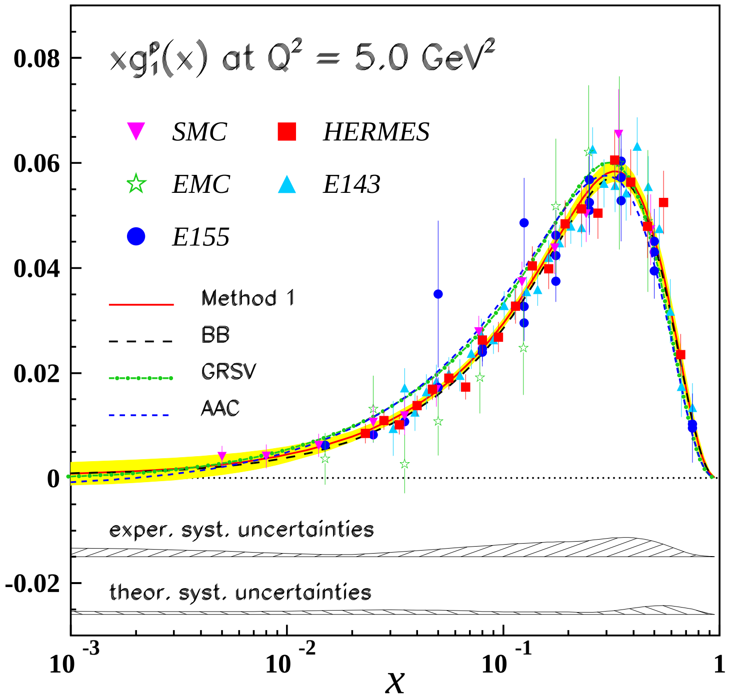
- all published data from SLAC, EMC, SMC and Hermes used
- update of AAC00 analysis, same method
- main difference: inclusion of E155 proton data

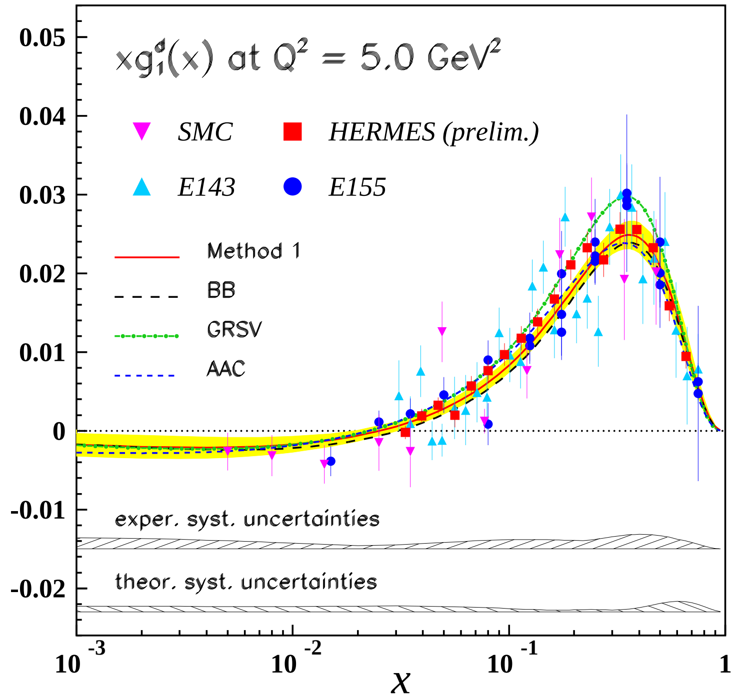
AAC03 PDFs

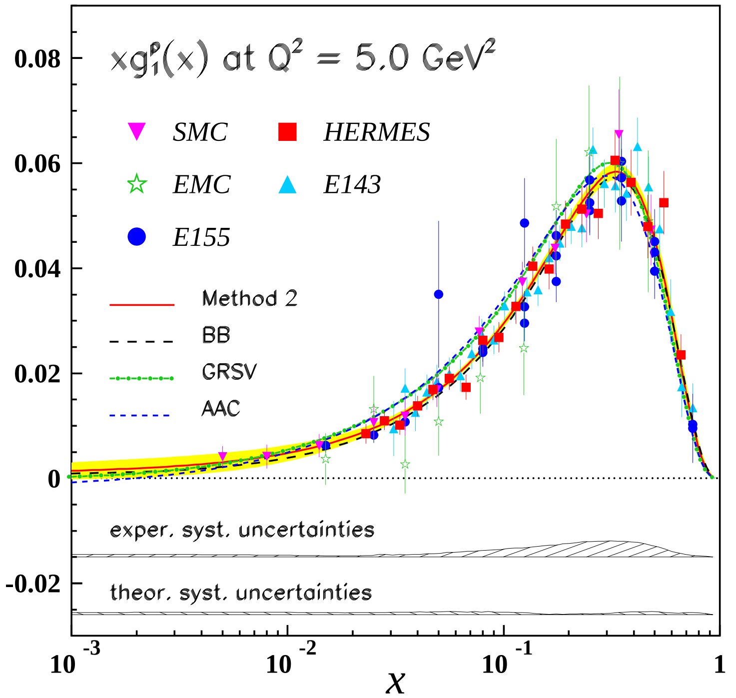


- Valence quark distributions well determined
- Antiquark uncertainity reduced
- $\Delta g(x)$ not determined
- $\Delta\Sigma = 0.213 \pm 0.138$ at $Q^2 = 1$ GeV^2

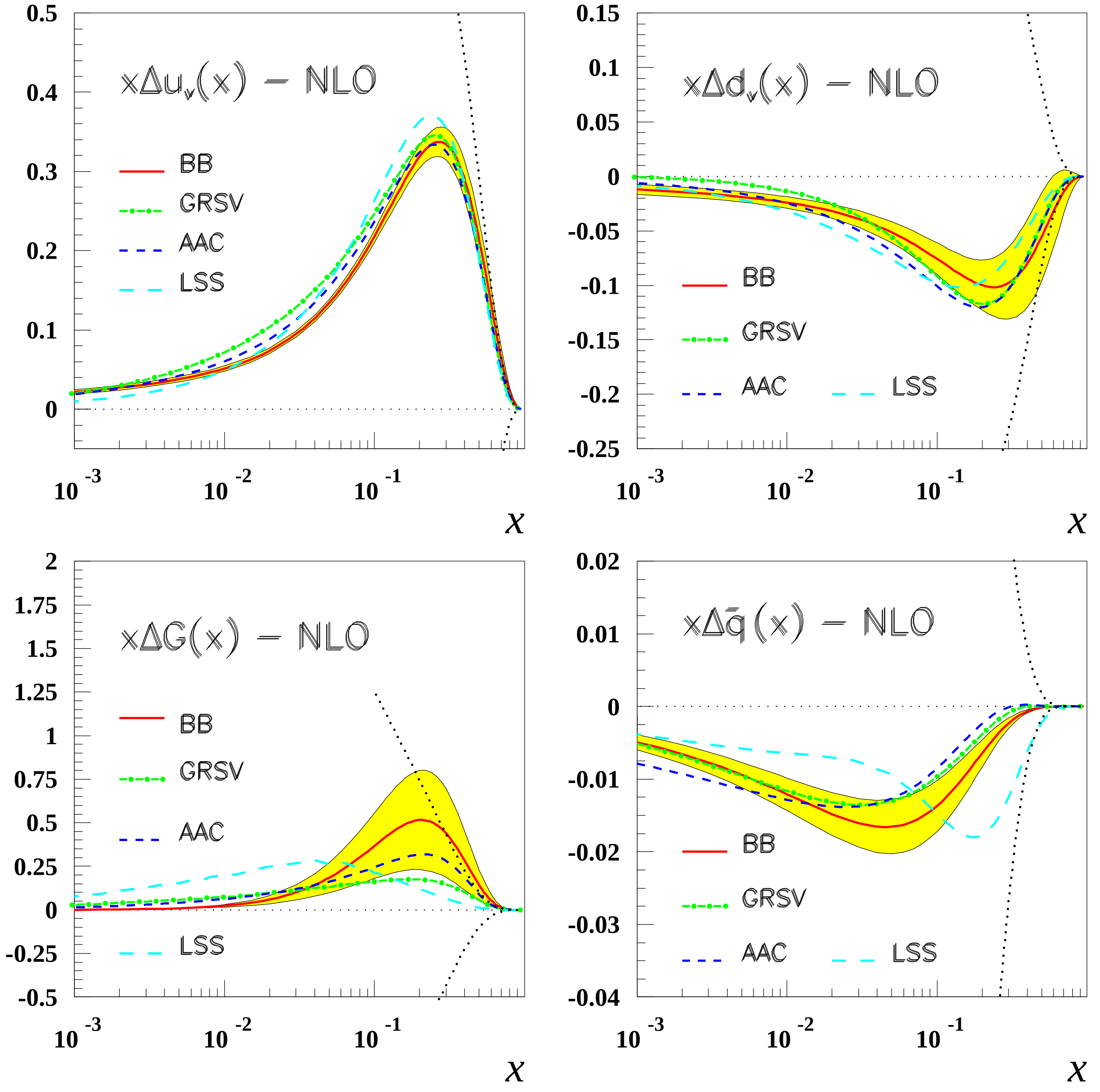
NLO QCD FITS OF g_1







POLARISED PDFs



- BB: Blümlein, Böttcher, hep-ph/0203155
- LSS: Leader et el., hep-ph/0111257
- GRV: Glück et al., hep-ph/0011215
- AAC: Goto et al., hep-ph/0001046

SUM RULES

Moments

- $a_q = \int_0^1 \Delta q(x) \mathrm{d}x$
  a_{q} contribution of quarks q to nucleon spin
- $\Gamma_1 = \int_0^1 g_1(x) \mathrm{d}x$: $\Gamma_1^{\mathrm{p,n}} = \pm\frac{1}{12} a_3 + \frac{1}{36} a_8 + \frac{1}{9} a_0$
- Axial charges
  $a_3 = a_{\mathrm{u}} - a_{\mathrm{d}}$
  $a_8 = a_{\mathrm{u}} + a_{\mathrm{d}} - 2a_{\mathrm{s}}$
  $a_0 = a_{\mathrm{u}} + a_{\mathrm{d}} + a_{\mathrm{s}}$ Contribution of all quarks to nucleon spin

Sum rules

- Bjorken sum rule
  $\mathrm{SU}(2)_{\mathrm{F}}$ symmetry: $\Delta u_{\mathrm{n}} = \Delta d_{\mathrm{P}}$ etc.
  $a_3 = |g_{\mathrm{A}}/g_{\mathrm{V}}| = 1.267 \pm 0.0035$ neutron decay

$$\Gamma_1^{\mathrm{p}} - \Gamma_1^{\mathrm{n}} = \frac{1}{6} a_3 = \frac{1}{6} |g_{\mathrm{A}}/g_{\mathrm{V}}|^{\mathrm{n}\to\mathrm{p}}$$

- Ellis-Jaffe sum rule
  $\mathrm{SU}(3)_{\mathrm{F}}$ symmetry in baryon octett
  $a_8 = 3F - D = 0.585 \pm 0.025$ hyperon decays

$$a_0 = a_8 \sim 0.6 \quad \text{with } a_{\mathrm{s}} = 0$$

QCD CORRECTIONS

- **Structure functions**

$$g_1^{\mathrm{p,n}}(x,Q^2) = \frac{1}{2}(\pm\Delta q_3(x,Q^2) + \frac{1}{3}\Delta q_8(x,Q^2)) \otimes C^{\mathrm{NS}}$$
$$+\frac{1}{9}(\Delta\Sigma(x,Q^2) \otimes C^{\mathrm{S}} + \Delta g(x,Q^2) \otimes 2n_{\mathrm{f}}C^{\mathrm{G}}$$

- **Axial charges**

$$a_0(Q^2)C_1^{\mathrm{S}}(Q^2) = \Delta\Sigma(Q^2)C_1^{\mathrm{S}}(Q^2) + \Delta G(Q^2)2n_{\mathrm{f}}C_1^{\mathrm{G}}(Q^2)$$

- **Moments**

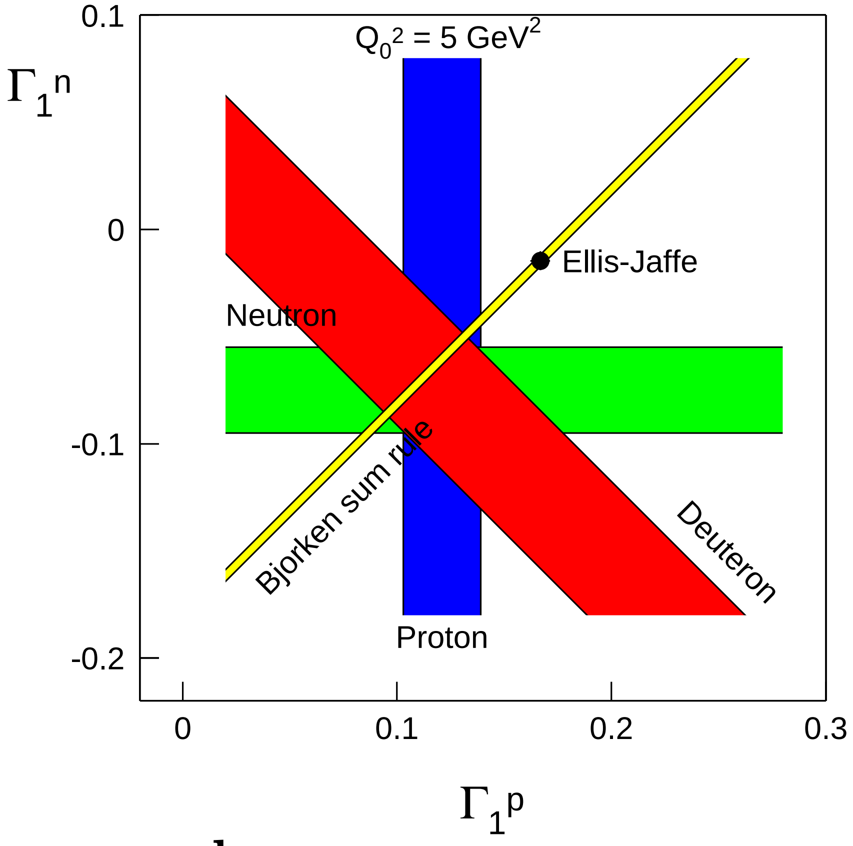
$$\Gamma_1^{\mathrm{p,n}}(Q^2) = \frac{1}{12}(\frac{a_8}{3} \pm a_3)\ C_1^{\mathrm{NS}}(Q^2) + \frac{1}{9}\ a_0(Q^2)\ C_1^{\mathrm{S}}(Q^2)$$

- **Bjorken sum rule**

$$\Gamma_1^{\mathrm{p}} - \Gamma_1^{\mathrm{n}} = \frac{1}{6}\,|g_{\mathrm{A}}/g_{\mathrm{V}}| \cdot C_1^{\mathrm{NS}}(Q^2) = \frac{1}{6}a_3 \cdot C_1^{\mathrm{NS}}(Q^2)$$

SUM RULES AT $Q_0^2 = 5$ GeV^2

- **Moments**



- **Bjorken sum rule**

$$\mathbf{\Gamma_1^p - \Gamma_1^n \; = \; 0.176 \; \pm 0.003 \; \pm 0.007}$$

(Prediction: $\Gamma_1^p - \Gamma_1^n = 0.181 \pm 0.005$)

- **First moment of $\Delta g(x)$**

$$\mathbf{a_g} = 1.6 \; \pm 0.8 \text{ (sta)} \;\; \pm 1.1 \text{ (sys)}$$

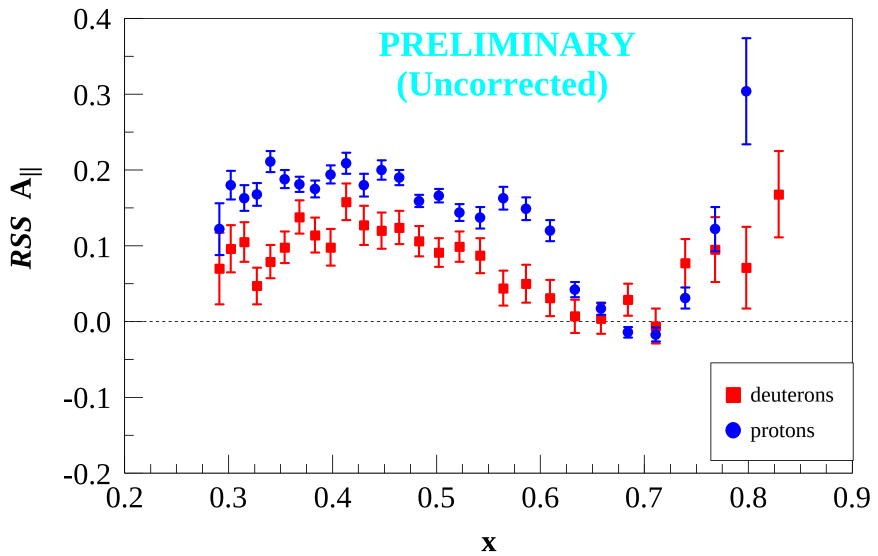
- **Axial charge a_0**

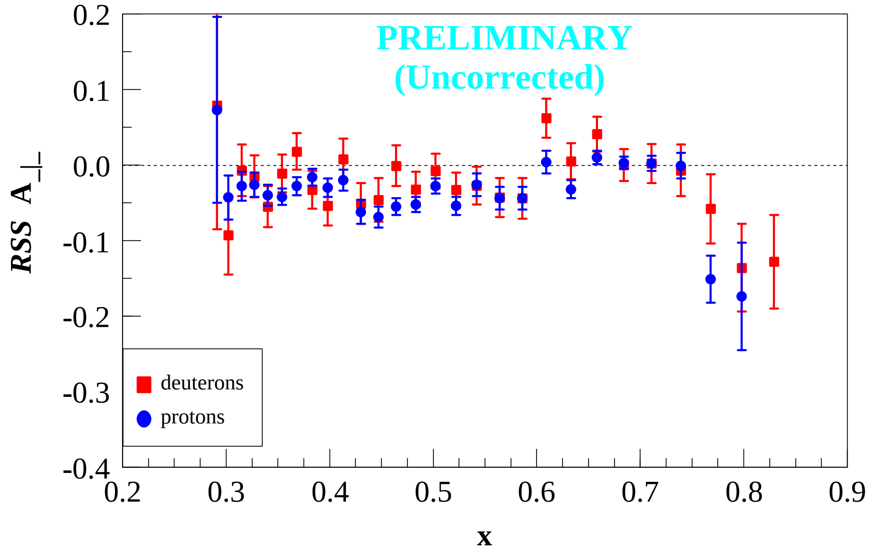
$$\mathbf{a_0} = 0.23 \; \pm \; 0.04 \text{ (sta)} \;\; \pm \; 0.6 \text{ (sys)}$$

(E155 NLO analysis in $\overline{\text{MS}}$ scheme)

NEW MEASUREMENTS

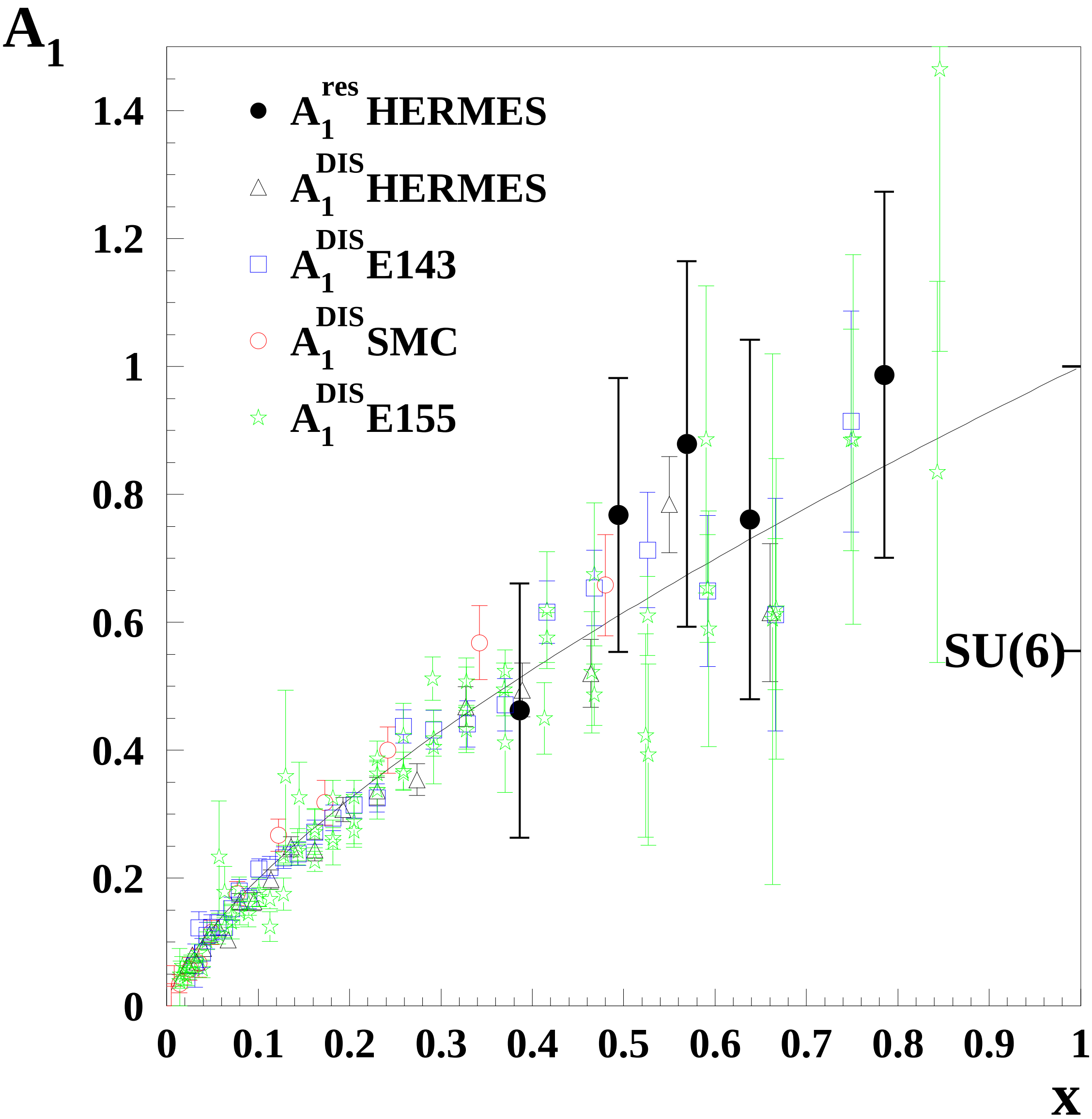
- **JLAB E-01-006 (RSS)**
  A_1 and A_2 for p and d at $Q^2 \approx 1.3$ GeV^2 for
  $0.8 \leq W \leq 2GeV$
  preliminary results for experimental asymmetries in resonance region





TEST OF LOCAL DUALITY

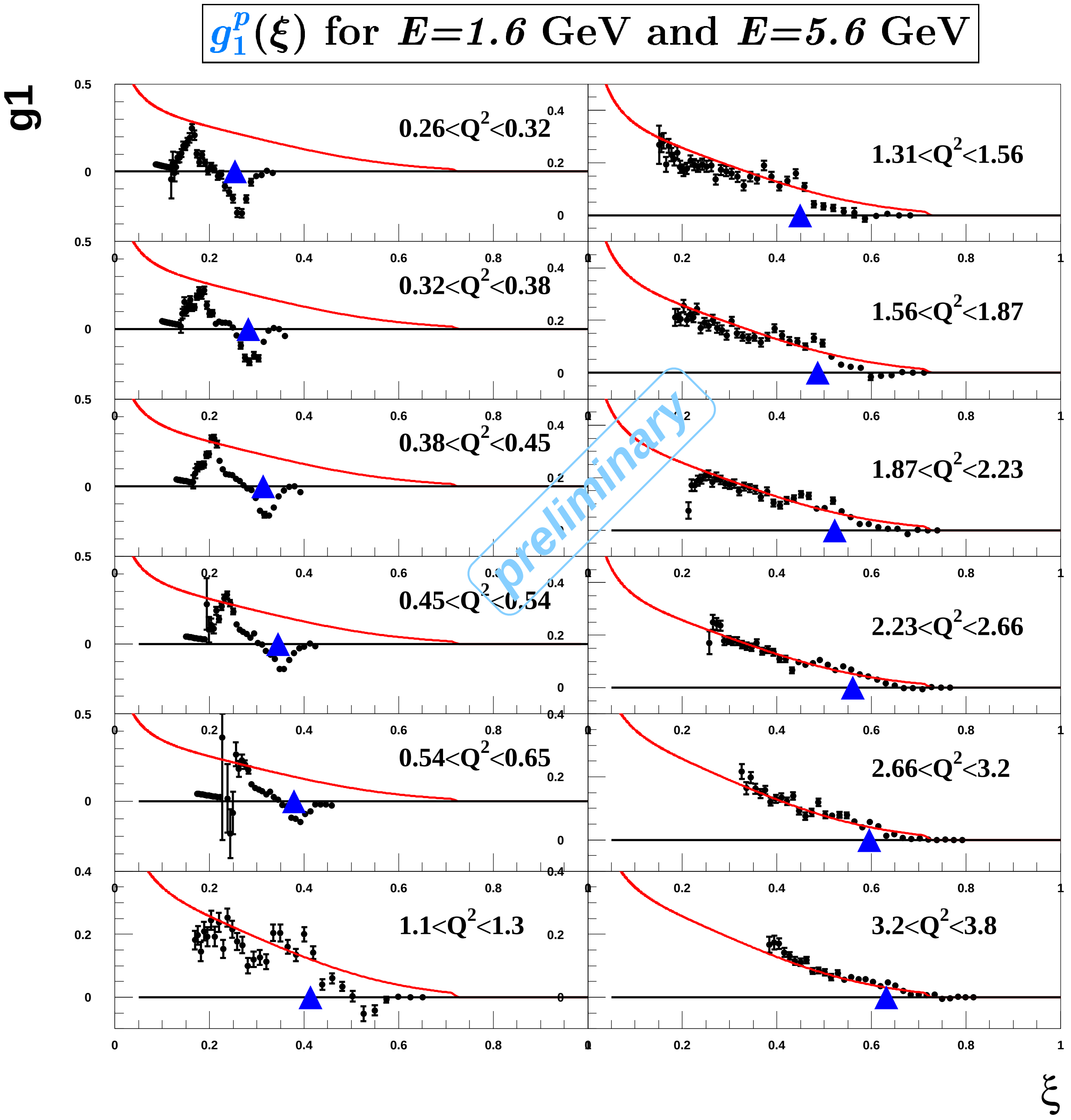
- Quark-hadron duality already observed for unpolarised structure functions
- Average behaviour in resonance region in agreement with DIS data



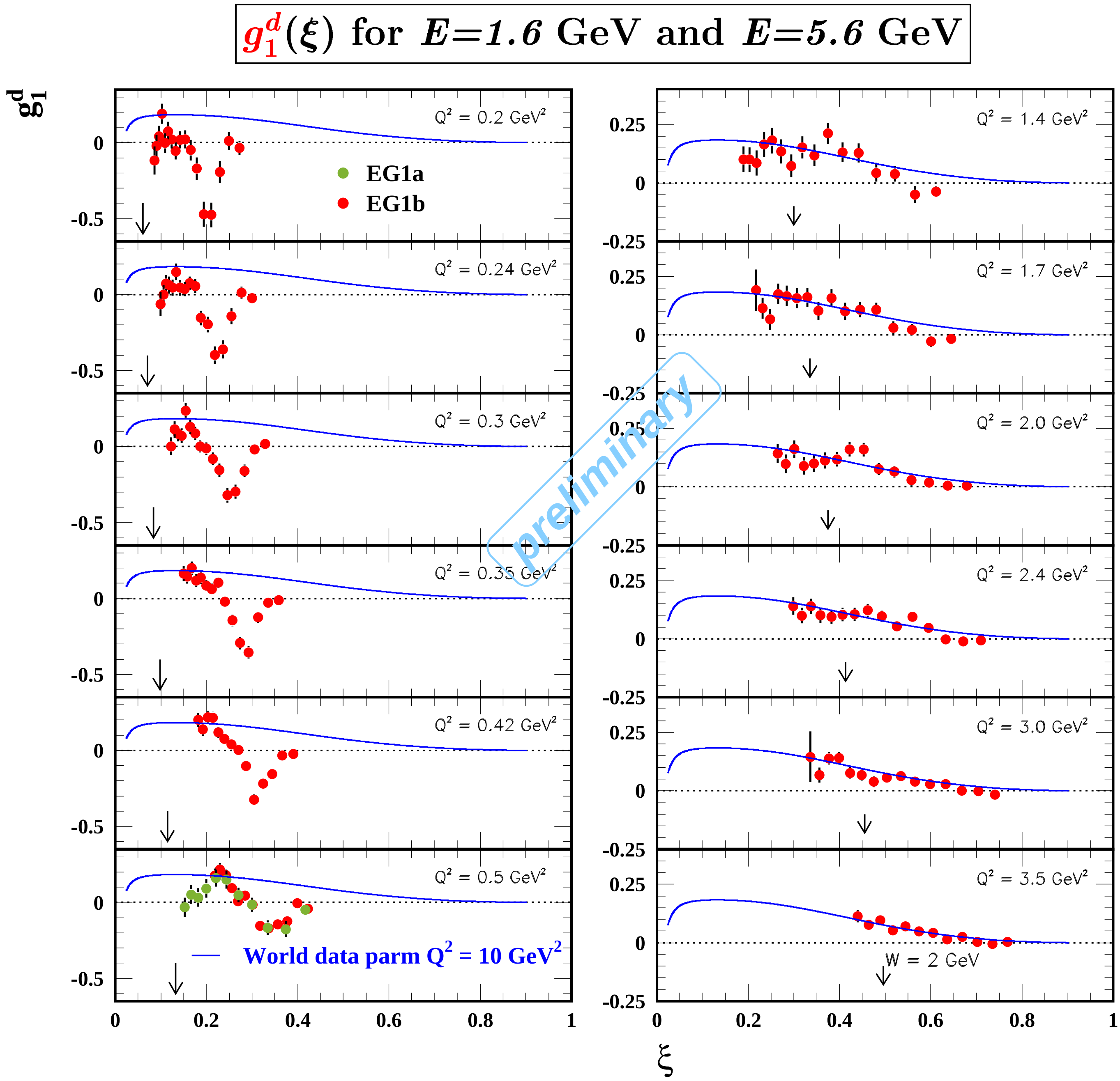
- Detailed test of duality with CLAS data

DUALITY FOR g_1^p

- **JLAB CLAS EG1**
  high statisctics g_1 for p and d in resonance region, Q^2 dependence of Γ_1^p



DUALITY FOR g_1^d



- Nachtmann variable $\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$
- Curves: global fit to world g_1 data at $Q^2 = 10$ GeV^2
- Duality valid for p and d (excluding Δ region)

SUMMARY

- Inclusive measurements on p,d,n targets
- Precise data in DIS range and resonance region
- Good description by NLO QCD
- Δg, Δs hardly constrained
- Bjorken sum rule verified to few %
- Quark-hadron duality studied
- New data to come (COMPASS, JLAB)