Produktion neutraler Mesonen in Proton-Proton Streuung bei COMPASS

HK 28.2

Tobias Weisrock
for the COMPASS collaboration

Johannes Gutenberg-Universität Mainz

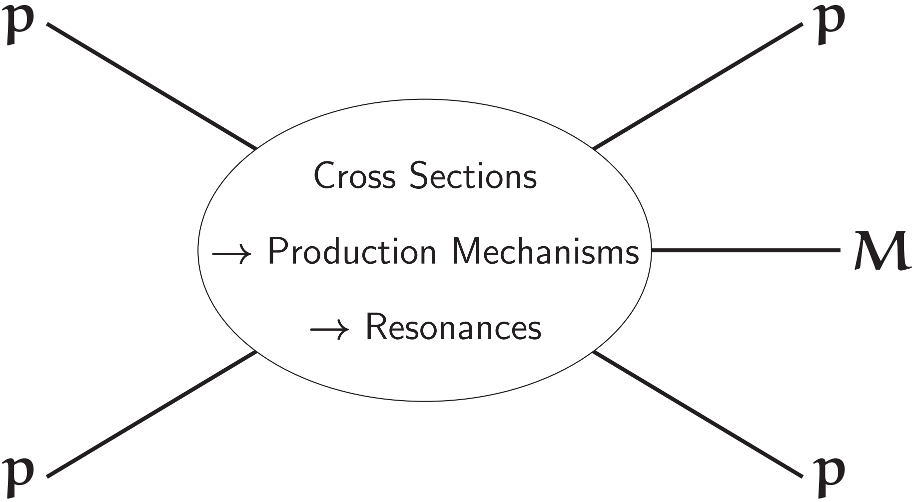
DPG-Frühjahrstagung Heidelberg
24. März 2015

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

bmb+f - Förderschwerpunkt COMPASS Großgeräte der physikalischen Grundlagenforschung

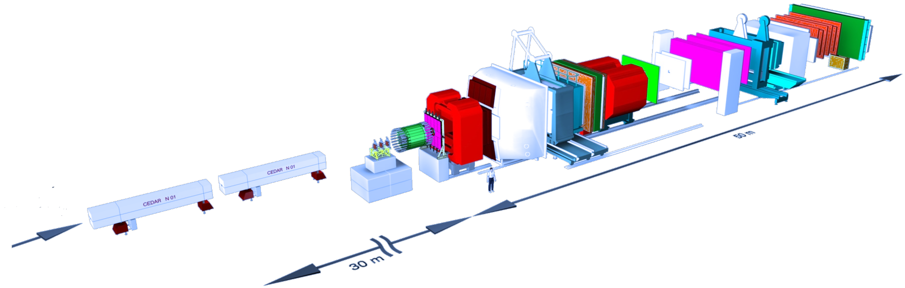
COMPASS

Motivation



The COMPASS Experiment

- **CO**mmon **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy
- Located at SPS at CERN



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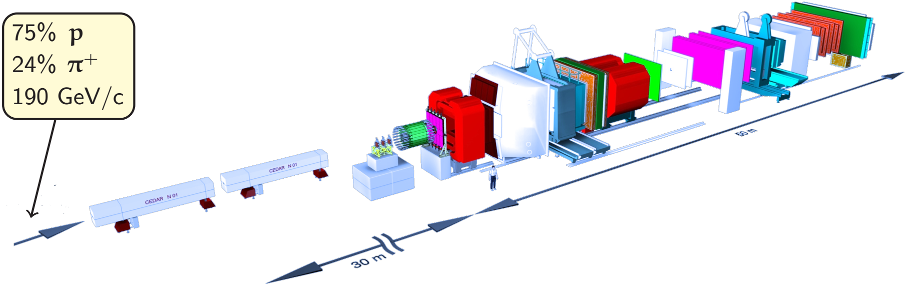

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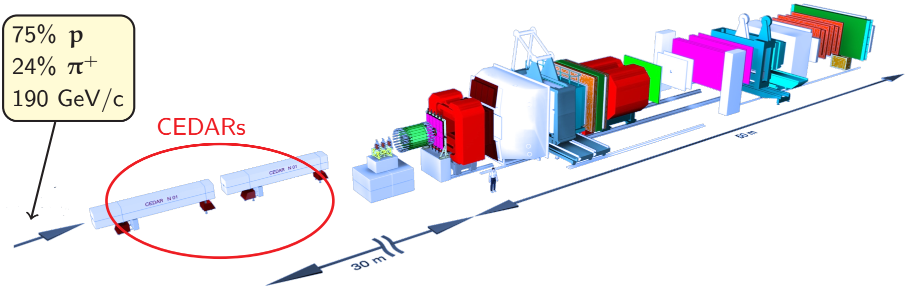

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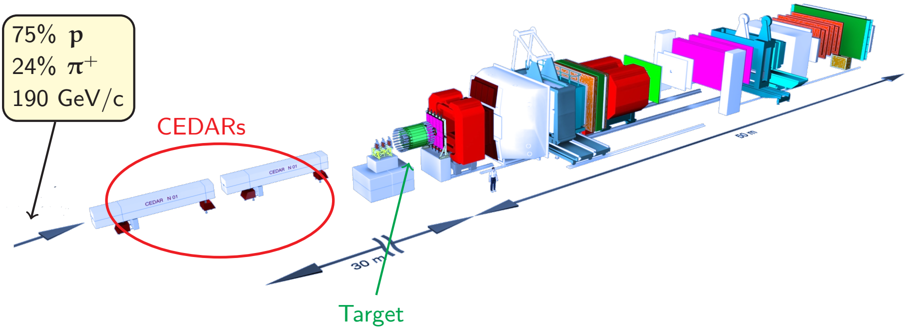

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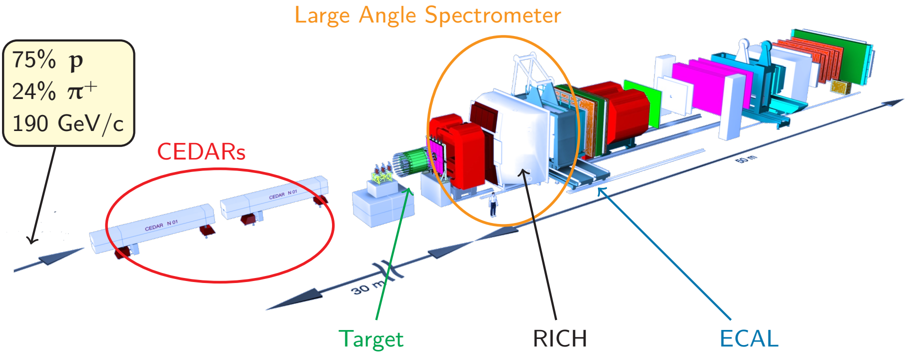

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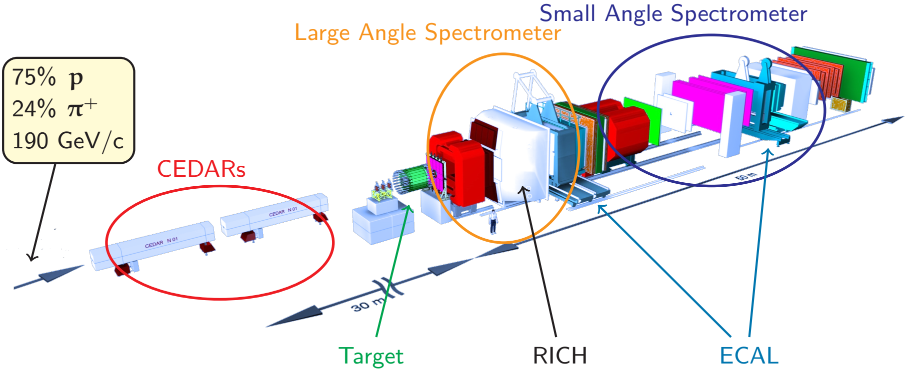

Introduction

- 2009 data taking
- 190 GeV/c proton beam on liquid hydrogen target
- diffractive dissociation
- Different channels investigated:
  - $pp \rightarrow p_f \pi^0 p_{recoil}$
  - $pp \rightarrow p_f \eta p_{recoil}$
  - $pp \rightarrow p_f \omega p_{recoil}$
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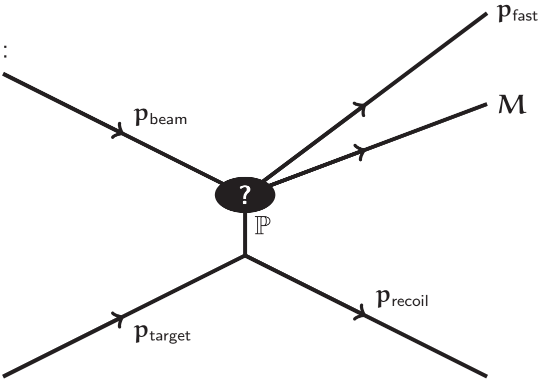

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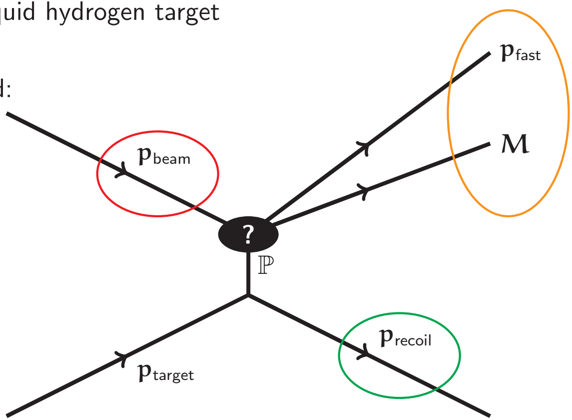

Event Selection

Basic Cuts

- minimum bias trigger
  - incoming beam + recoiling proton
- exactly 1 primary vertex reconstructed inside the target
- identified incoming proton
- 1 reconstructed recoil proton
- $\approx 4 \times 10^9$ events at this stage

Selection of π^0/η

- 1 outgoing charged particle with positive charge
- 2 ECAL clusters
  - Combined to a π^0 or η

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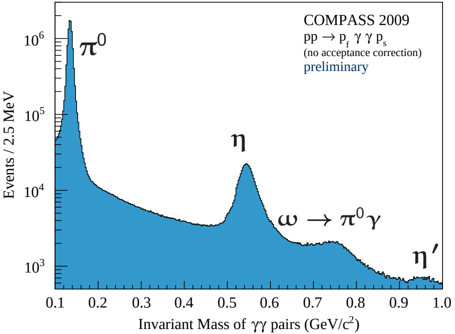

Selection of π^0/η

- 1 outgoing charged particle with positive charge
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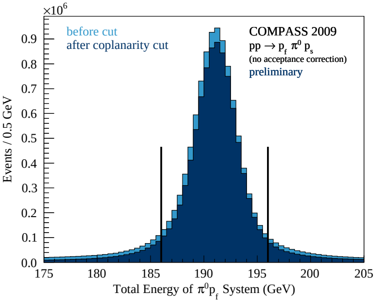
Event Selection

Exclusivity and Coplanarity

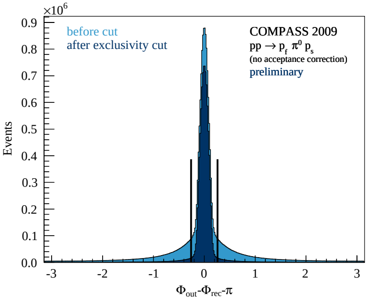
Exclusive events selected by 2 cuts:

- energy sum of outgoing system around peak value (exclusivity)
- azimuthal angles of outgoing system and recoil proton differ by π (coplanarity)

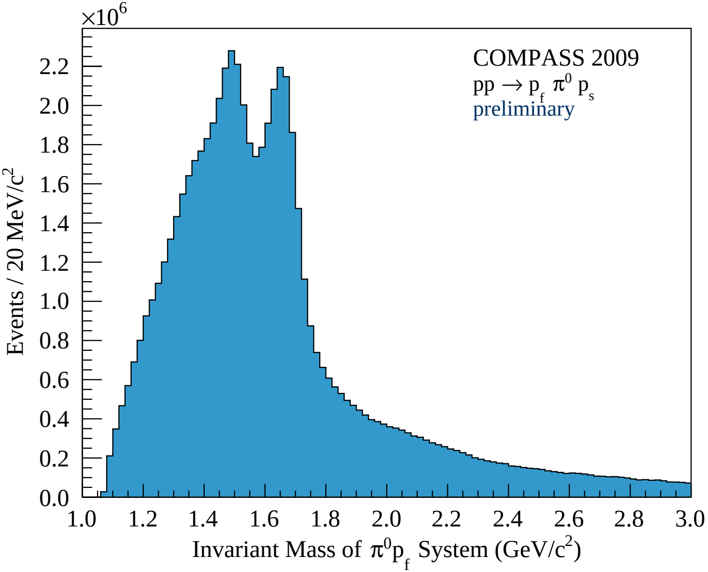
Exclusivity



Coplanarity



Goal: Understand $\mathbf{p}\boldsymbol{\pi}^0$ Mass Spectrum



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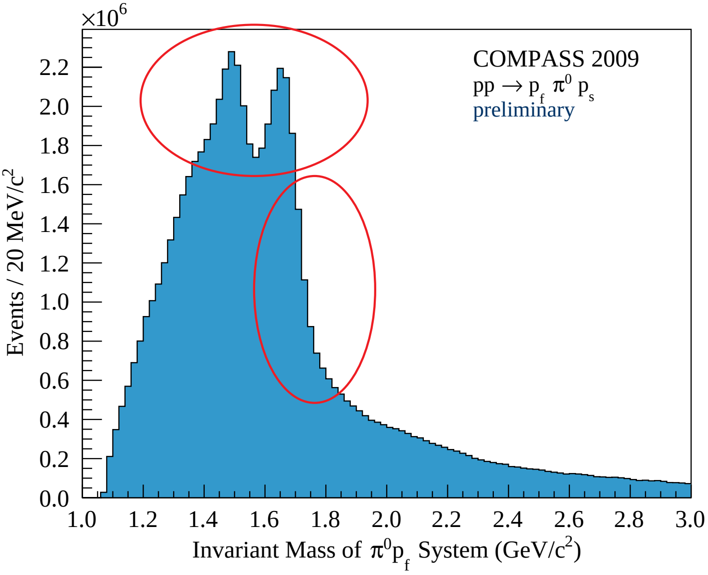
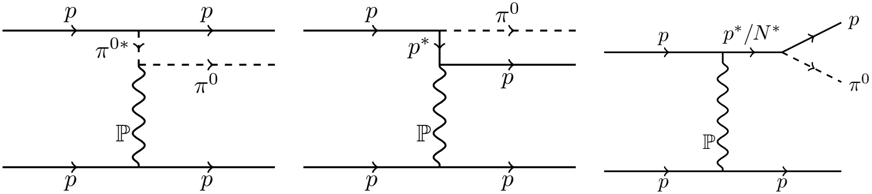


Production Mechanisms in $\mathbf{pp \to pp\pi^0}$

Main process: diffractive bremsstrahlung (Drell-Hiida-Deck)

- pion exchange (central production)
- proton exchange
- direct production (+ resonances)

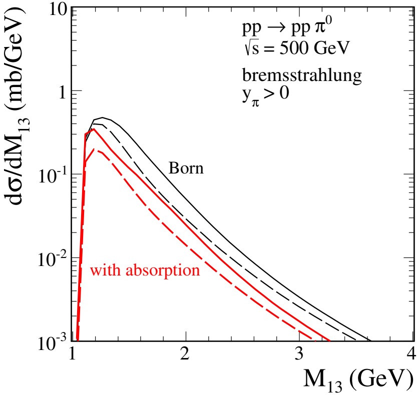


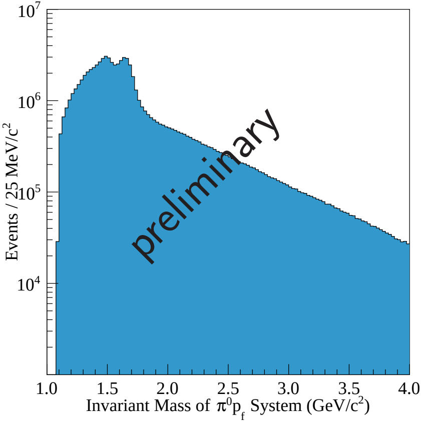
Further processes:

- $\gamma\gamma$ fusion
- $\gamma\omega^*$ fusion
- odderon exchange

[Phys.Rev. D87(7):074037, 2013, hep-ph/1303.2882]

Comparison of Calculation and Data

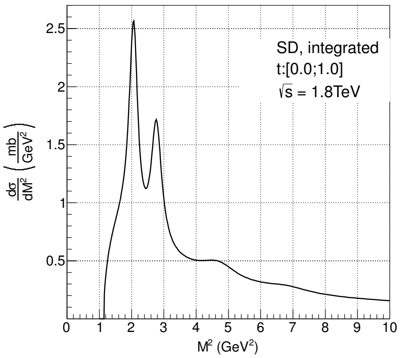


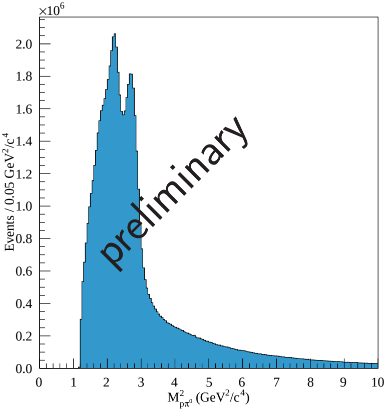


- exponential decline predicted by theory calculation
- resonances on top of background
  - → not included in calculation

Resonances

- calculation of resonances (N1440, N1680, N2220, N2700) based on LHC data
- shape of mass spectrum similar to data





[Phys.Atom.Nucl. 77(12):1463-1474, 2014, hep-ph/1211.5841]

Partial Wave Analysis

Last Year: Parametrise only resonance decay

Towards a Partial Wave Analysis

- Do not describe the full process
- Only investigate two-body decay into $\mathbf{p\pi^0}$
- Fit in bins of the invariant $\mathbf{p\pi^0}$ mass

Intensity (fit function)

$$\mathcal{I} = \sum_{\epsilon}\sum_{\lambda}\left|\sum_{k} T_k^{\epsilon} A_k^{\epsilon,\lambda}(\theta,\phi;m_{p\pi^0})\right|^2$$

with (complex) strength $\mathbf{T_k}$, reflectivity $\epsilon = \pm\mathbf{i}$, proton helicity $\lambda = \pm\frac{1}{2}$

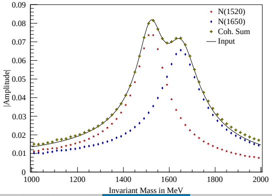
Partial Wave Amplitude

$$A_k^{\epsilon,\lambda}(\theta,\phi;m_{p\pi^0}) = \sqrt{2L+1}(L\ 0, \tfrac{1}{2}\ \lambda|J\ M)D_{M\lambda}^{J,\epsilon\ *}(\phi,\theta,0)F_L(q)$$

with Blatt-Weisskopf barrier factor $\mathbf{F_L(q)}$

Testing the fit

- Produce angular distributions in mass bins
  - include two Breit-Wigner peaks in mass spectrum
  - include phase shift between the peaks
- Run the fit over the produced distributions



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Baryon Spectroscopy at COMPASS

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Tobias Weisrock (JGU Mainz) 17. März 2014 9 / 12

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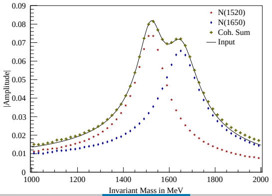

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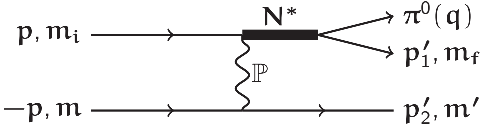
Today:

- large non-resonant background
- cannot be described in simple model
- create model for full process (formation and decay of resonance)
- include diffractive background

New Model

proton-proton scattering in CMS

$$p(E, \vec{p}) + p(E, -\vec{p}) \longrightarrow p(E_1', \vec{p}_1') + p(E_2', \vec{p}_2') + \pi^0(\omega_\pi, \vec{q}_\pi)$$



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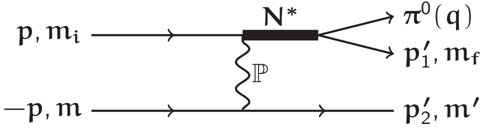

5 independent kinematic variables:

- invariant mass $\omega_{\pi p}$ of pion and fast proton p_1'
- direction of the π-p_1'-system ($\equiv$ recoil proton p_2') $\to \Omega = (\theta, \varphi)$
- direction of pion in the GJ-frame $\to \Omega_\pi^* = (\theta_\pi^*, \varphi_\pi^*)$

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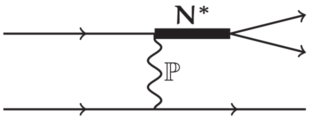

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cross section:

$$\frac{d\sigma}{d\omega_{\pi p}\, d\Omega\, d\Omega_\pi^*} = \frac{1}{(2\pi)^5} \frac{2M_N^4 p_2' q}{E^2 p} \sum_{S_i, M_i, S_f, M_f} |T_{S_i, M_i, S_f, M_f}|^2$$

with total initial/final spin
$S_{i,f} = 0, 1$ and $M_{i,f} = -S_{i,f}, \ldots, S_{i,f}$

Amplitude

$$T_{S_i,M_i,S_f,M_f}(\vec{p},-\vec{p},\vec{p}'_1,\vec{p}'_2) = \sum_{N^*(J^{\mathcal{P}})} \sum_{m_i,m_f,m,m',m_\pi,m_{\mathbb{P}},M_J}$$

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Clebsch-Gordan coefficients for single spin couplings
pomeron couplings to pp and pN^*, unknown constants
resonance shape in $p\pi$-mass $\longrightarrow$ free parameters
pomeron propagator
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Background not yet included in model

Conclusion and Outlook

- COMPASS has large datasets single meson production in $\mathbf{pp}$ scattering
- different channels accessible
- shape of invariant mass spectrum understood qualitatively
- partial wave analysis model developed for full process
- background parametrisation not yet included
- details in my thesis (summer '15)

Thank you for your attention

Alternative Approach

- Meeting with baryon spectroscopy experts in Mainz
- Include production mechanism in amplitude calculations

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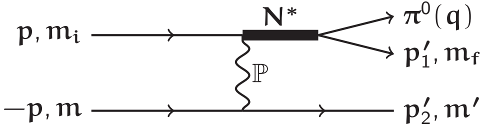

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The angular momentum of pion and pomeron is fixed for a given $J^{\mathcal{P}}$:

$N^*(J^{\mathcal{P}})$	$S_{11}(\frac{1}{2}^-)$	$P_{11}(\frac{1}{2}^+)$	$P_{13}(\frac{3}{2}^+)$	$D_{13}(\frac{3}{2}^-)$	$D_{15}(\frac{5}{2}^-)$	$F_{15}(\frac{5}{2}^+)$
L_π	0	1	1	2	2	3
$L_{\mathbb{P}}$	1	0	2	1	3	2

spin projections m_π and $m_{\mathbb{P}}$ are free parameters

Amplitude

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vertex form factor $F(\omega_{\pi p}) = \frac{\Lambda^4}{\Lambda^4-(\omega_{\pi p}^2-M_{N^*}^2)^2}$ with $\Lambda = 1.3$ GeV

πNN^* coupling constant $f_{\pi NN^*}$ (free parameter)

resonance propagator $G_{N^*}(\omega_{\pi p}) = \frac{1}{\omega_{\pi p}-M_{N^*}+\frac{i}{2}\Gamma_{N^*}(\omega_{\pi p})}$

$N^* \to \pi N$ partial width $\Gamma_{N^*}(\omega_{\pi p}) = \frac{f_{\pi NN^*}^2}{4\pi}\frac{F^2(\omega_{\pi p})}{\omega_{\pi p}}\frac{2M_N}{M_\pi^{2L_\pi}}\frac{L_\pi!}{(2L_\pi+1)!!}q^{2L_\pi+1}$

Amplitude

$$T_{S_i,M_i,S_f,M_f}(\vec{p},-\vec{p},\vec{p}'_1,\vec{p}'_2) = \sum_{N^*(J^{\mathcal{P}})} \sum_{m_i,m_f,m,m',m_\pi,m_{\mathbb{P}},M_J}$$

$$\left(\frac{1}{2}m_i,\frac{1}{2}m\middle|S_iM_i\right)\left(\frac{1}{2}m_f,\frac{1}{2}m'\middle|S_fM_f\right)\left(\frac{1}{2}m_i,L_{\mathbb{P}}m_{\mathbb{P}}\middle|JM_J\right)\left(\frac{1}{2}m_f,L_\pi m_\pi\middle|JM_J\right)$$

$$\frac{f_{\mathbb{P}NN^*}f_{\mathbb{P}NN}}{M_\pi^{L_\pi+L_{\mathbb{P}}}} \times F(\omega_{\pi p})f_{\pi NN^*}G_{N^*}(\omega_{\pi p}) \times G_{\mathbb{P}}(t) \times (-1)^{m_{\mathbb{P}}}p^{[L_{\mathbb{P}}]}_{-m_{\mathbb{P}}}q^{[L_\pi]}_{m_\pi}$$

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pomeron propagator parameterised through Regge trajectory:

$$G_{\mathbb{P}}(t) = \left(\frac{s}{s_0}\right)^{\alpha(t)-1}\frac{\pi\alpha'}{\sin\left(\pi\alpha(t)\right)}\frac{e^{-i\pi\alpha(t)}}{\Gamma\left(\alpha(t)\right)}$$

with

$$s_0 = 1\text{ GeV}$$

$$s = 2\sqrt{\vec{p}'^2_1 + M_N^2} \quad t = (E(N^*) - E)^2 - (\overrightarrow{N^*} - \vec{p})^2$$

$$\alpha(t) = \alpha_0 + \alpha' t, \quad \alpha_0 = 1.08, \ \alpha' = 0.25$$

Amplitude

$$T_{S_i,M_i,S_f,M_f}(\vec{p},-\vec{p},\vec{p}_1',\vec{p}_2') = \sum_{N^*(J^{\mathcal{P}})} \sum_{m_i,m_f,m,m',m_\pi,m_{\mathbb{P}},M_J}$$

$$\left(\frac{1}{2}m_i,\frac{1}{2}m\middle|S_iM_i\right)\left(\frac{1}{2}m_f,\frac{1}{2}m'\middle|S_fM_f\right)\left(\frac{1}{2}m_i,L_{\mathbb{P}}m_{\mathbb{P}}\middle|JM_J\right)\left(\frac{1}{2}m_f,L_\pi m_\pi\middle|JM_J\right)$$

$$\frac{f_{\mathbb{P}NN^*}f_{\mathbb{P}NN}}{M_\pi^{L_\pi+L_{\mathbb{P}}}} \times F(\omega_{\pi p})f_{\pi NN^*}G_{N^*}(\omega_{\pi p}) \times G_{\mathbb{P}}(t) \times (-1)^{m_{\mathbb{P}}} P^{[L_{\mathbb{P}}]}_{-m_{\mathbb{P}}} q^{[L_\pi]}_{m_\pi}$$

Angular dependence for momentum $\vec{Q} = (Q, \theta_Q, \varphi_Q)$:

$$Q_M^{[L]} = \sqrt{\frac{4\pi L!}{(2L+1)!!}} Q^L Y_{LM}(\theta_Q, \varphi_Q)$$

$\vec{P}$ is the relative momentum of pomeron und beam:

$$\vec{P} = \frac{\vec{p}_{\mathbb{P}}E - \vec{p}E_{\mathbb{P}}}{E + E_{\mathbb{P}}}$$

$$\vec{p}_{\mathbb{P}} = \overrightarrow{N^*} - \vec{p} \quad E_{\mathbb{P}} = E(N^*) - E$$